COURNOT EQUILIBRIUM IN A MODEL OF HARDWARE AND SOFTWARE MANUFACTURERS' INTERACTION

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ABSTRACT: A model of interaction between hardware vendors, Intel and AMD, and developers of Windows and Linux operating systems is suggested. Intel and AMD both maximize profits forming a traditional oligopoly, while Microsoft and the community of Linux developers form a mixed duopoly, in which only the first party maximizes its profit. We consider a Cournot situation, when each of the profit-maximizing suppliers sets the price based on available market information on other players' products prices in the previous time moment, and assuming the cross-price elasticities to zero. At the Cournot equilibrium, an Intel-based PC running Windows is 5 times more expensive than AMD-based PC running Linux; an Intel CPU costs 2 times more than AMD processor; Windows license is 1,5 times more expensive than Intel processor; and the profit of Intel is 4 times greater than the profit of AMD, while Microsoft has just 12,5% greater profit than Intel.

Key words: complementors, complements, co-opetition, Cournot equilibrium, pricing.

JEL codes: C72, D43, K21, L13, L15, M21.

Introduction

Over the recent years, an increasing large number of industries have evolved from vertical to horizontal integration, where some firms design and manufacture components which are assembled by other firms for the final customers. In these horizontal industries, firms are 'complementors' rather than customers, suppliers, or competitors. IT industry demonstrates the most striking examples of such an organization. There are suppliers of hardware components (processors, memory modules, motherboards, video cards, monitors, drives, etc.), suppliers of software (operating systems, office suites, etc.), and assemblers of computers providing the market with servers and workstations (usually with preinstalled software).

CPUs could be made by *Intel* or *AMD*, PCs could be assembled by *ASUS*, *Dell*, *Hewlett Packard*, *IBM* and others, on the same computer one of the two operating systems can be installed (*Microsoft Windows* or *Linux*), and various applications can work under different operating systems (e. g., *Microsoft Office* and *OpenOffice*).

The horizontal integration of IT industry is linked to setting up an open standard for *IBM PC* in 1980. As a result, there was a deep specialization of component manufacturers, assemblers, and software developers. In particular, the *IBM*'s decision of choosing *Intel* and *Microsoft* as manufacturers of CPUs and operating systems as the key PC components has led to *Intel* and *Microsoft* domination at the PC market for almost 30 years (in contrast to *IBM*, which has lost its strategic positions in this market).

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According to Casadesus-Masanell, Nalebuff and Yoffie (2007a), in 2007 more than 80% of the PCs worldwide were shipped with an *Intel* CPU running *Microsoft Windows* operating system while there are a lot of producers of other PC components (motherboards, memory modules, drives, monitors, etc.) in this market.

As it was demonstrated by Yoffie, Casadesus-Masanell and Mattu (2004), the combined profit of *Intel* and *Microsoft* during most years in the 1990s exceeded the total profit of the entire world PC industry.

In 2004, for example, *Intel* and *Microsoft* earned over $15B in net profits while the three largest assemblers (*Dell*, *HP* and *IBM*) made roughly just $2.5B in profits from their PC operations. IBM alone lost over $1B in PCs in 1998, and another $1B between 2001 and 2004. Only *Dell* made material profits in the PC industry at that time. For more detailed information on market dynamics see Yoffie, Casadesus-Masanell and Mattu (2004).

This gives us the reason to assume *Microsoft* and *Intel* the key strategic players in the market of PCs that have a direct impact on the final product price (unlike the manufacturers of the other components).

Unlike the microprocessor market, where the main player (*Intel*), and its closest competitor (*AMD*), and other manufacturers seek to maximize their profits, the modern software market is characterized by the asymmetry of suppliers' interests, some of the software suppliers are maximizing their profits from the sale of licenses for their products (like *Microsoft*), while other software developers are distributing their products for free, often even with open source code.

For example, for-profit manufacturers had a monopoly in the server operating systems market 15 years ago, because the users did not trust non-commercial software that did not guarantee quality, reliability and security; now the commercial software product *Microsoft Windows* and its non-commercial competitor *Linux* each have approximately 40% of the server operating systems market. In contrast to Microsoft, the *Linux* developers community distributes this operating system for free and with open source code under GNU GPL license)

The competition in the software market essentially differs from a competition in the markets of traditional material goods (including CPUs) due to the special features of software as a good, first of all, due to the absence of rarity.

At the moment all the software users are choosing between the three options:

- to buy licenses and use the commercial proprietary software (e. g. *Microsoft Windows* as an operating system, *Microsoft Office* as an office suite, *Microsoft SQL Server* as a database server, *Microsoft Internet Information Server* as a web server, etc.);
- to use free or open source software (e. g. *Linux* as an operating system, *OpenOffice* as an office suite, *MySQL* as a database server, *Apache* as a web server, etc.);
- to pirate (i. e. to use proprietary software without buying licenses).

These three options correspond to the following three types of software market players:

profit-maximizers (for example, *Microsoft*);

- non-for-profit players (for example, *Linux* team);
- pirates.

Correspondingly, the software developers try to determine the optimal way of income collecting:

- to sell the licenses for the use of their products;
- or to distribute the products for free and to collect incomes from sales of additional services.

As the real market shows, there is no unambiguous answer for the formulated questions. There are whole countries, using illegal copies of the proprietary software almost at 100%.

The number of non-commercial software users grows, and software developers that distribute their products with open source code receive stable incomes.

But the number of commercial software users remains high, allowing for-profit suppliers to receive a steady income from the sales of licenses.

Such a market structure requires new approaches to research methodology as well as to business development methodology.

Literature review

A.Cournot in the seminal book (1838) has considered the first model in mathematical economy for interactions between monopolists producing complementary products (manufacturers of copper and zinc that are combined to make a brass as a composite product).

The main result developed by Cournot using this model of complementary products suppliers' interaction is that suppliers will divide the profits equally non-depending on the relation of the components prices!

However, in the real IT market there is a competition both between hardware manufacturers (there are *Intel*-based and *AMD*-based servers and workstations in the market), and between of operating systems suppliers (*Microsoft Windows* and *Linux*).

While price competition between vertically differentiated goods as well price competition between complementors are each well understood in mathematical economics, but the combined case of competition between competing complements is investigated insufficiently and needs additional research.

McAfee, McMillan and Whinston (1989) studied game-theory model of packaging products into a bundle and obtained conditions when bundling is an optimal strategy for the suppliers. Developing this research Choi and Stefanadis (2001), and Nalebuff (2004) investigated a question on expediency of entering into the market with a composite product.

Brandenburger and Nalebuff (1996) have considered Intel and Microsoft as an example of players who co-operate and compete simultaneously, they even have introduced the term «Co-Opetition» for a designation of interaction of players of this kind.

Casadesus-Masanell and Yoffie (2006) suggested the game model for the situation of co-operation and competition of *Intel* and *Microsoft*, and as a result of research it has appeared that unlike Cournot model where both manufacturers divide the profits fifty-fifty, in this case the optimal strategy of Microsoft is to underprice in order to increase the client base, but Intel in reply to it should simply overprice and get the additional profit because an operating system is not on sale separately from a PC (see also the case study by Joffe, Kasadesus-Masanell and Mattu (2004).

Farrell and Katz (2000) have considered a situation, when the exclusive manufacturer of one component enters into the competitive market of the second component in order to reduce its price, and as a consequence, the price of a composite product. This model can be applied to Intel activities on motherboards manufacturing in addition to CPUs, but not to interaction of manufacturers of software and hardware.

Cheng and Nahm (2007) have considered Stakelberg's strategy in a situation of cooperation and competition of exclusive manufacturers of two components, each of which can be used as a part of a composite product, and separately. In the IT market using of one component (hardware or software) without another is impossible, and in addition there is a competition between manufacturers of components.

Chen, Nalebuff and Nalebuff (2006) have investigated competitive interactions in markets with one-way essential complements (first product is essential to the use of the second product, but can be used without the second product). Chen, Nalebuff and Nalebuff have applied this model to study the market of operating systems and applied software. They have shown that it is favourable to operating systems developer to enter into the competitive market of applied software with a competing version of application and to sell it at zero price. As a result, the existing competitors in the applied software market will be compelled to join the monopoly.

One of the recent steps in the duopoly theory was to combine the classic market duopoly theory with the demand-side learning and to extend this approach to a dynamic situation where the objectives of players are mixed rather than symmetric. This step was done by Cassadesus-Masanell and Ghemawat (2006) who have proposed a dynamic mixed duopoly model and applied this model to *Windows*/*Linux* competition dynamics.

Using the optimal control theory Casadesus-Masanell and Ghemawat (2006), and (with some extensions) Soloviev (2008a, 2008b, 2008c, 2009a) have obtained the conditions when *Linux* and *Windows* coexist in the market, and when one of the products is pushed out by another. The special focus in these models was given to a piracy of *Windows* and strategic contributions to *Linux* issues.

Among other works on IT economics we will note works of Katz and Shapiro (1985), Economides (1996), Yu (1998), Gawer and Cusumano (2002), and Soloviev (2009a), devoted to investigation of various network effects in the IT market.

It is necessary to mention the deep survey of the current state of the network economics applications to the IT market, with particular attention to such network effects, as scale effect, price discrimination, competition for a monopoly position and standards wars, in book by Varian, Farrell and Shapiro (2004).

Casadesus-Masanell, Nalebuff and Yoffie (2008) have presented a model of interaction of two competing hardware suppliers (*Intel* and *AMD*) with the exclusive operating systems manufacturer (*Microsoft*). This work represents, as a matter of fact, the first research of a competing complements. This paper considers competition between suppliers of complementing components (*Intel* and *Microsoft*), and between competing suppliers of similar components (*Intel* and *AMD*).

Soloviev (2009a, 2009b, 2009c) has extended the Casadesus-Masanell, Nalebuff and Yoffie (2008) model to interaction of two competing hardware suppliers (Intel and AMD) with two competing operational systems manufacturers, Microsoft corporation (the developer of proprietary Windows operating system), and non-commercial Linux operating system developers community.

Here we continue to study the problem of profit distribution among the suppliers of components in the IT market, started by Kasadesus-Masanell, Nejlbuff, Yoffie (2008) and Soloviev (2009a, 2009b, 2009c).

Assumptions

Let's discuss the basic assumptions of hardware and software suppliers' interaction model.

1º. The bundle of hardware (a PC) with preinstalled operating system is selling in the market.

There are Intel-based and AMD-based PCs in the market, each of which can be selling with one of two operating systems preinstalled (Windows or Linux).

Windows operating system is distributing by Microsoft corporation on a commercial basis by selling licenses, while Linux operating system is distributing by Linux developers community freely and free of charge.

The operating system is preinstalled on each PC before it enters the market, therefore we do not consider the possibility of a proprietary operating system illegal copies use.

Thus, the user selects one of four products:

- *Intel*-based PC running *Windows*;
- *Intel*-based PC running *Linux*;
- *AMD*-based PC running *Windows*;
- *AMD*-based PC running *Linux*.

2º. Nowadays, the *Windows*/*Linux* competition, especially in the netbooks segment, is growing because the *Windows* license costs more than 10% of the final product price for many models.

Although the *Windows* license price is positive, and *Linux* is distributed freely, both products co-exist on the market. It means that a consumer values *Windows* greater than *Linux*.

Intel-based PCs occupy the largest market share, so a consumer values Intel-based PC greater than *AMD*-based PC.

Thus, it is assumed that with other things being equal the consumer prefers an *Intel*-based PC to an *AMD*-based PC, and a PC running *Windows* to a PC running *Linux*.

It is supposed also that the difference in consumer value of different hardware is less, than a difference in consumer value of identical hardware with different operating systems.

3º. The demand functions for the combined products are linear.

4º. The user will buy the bundled product (a PC with an operating system) if and only if the consumer value of this product for this user exceeds its price.

Thus from two products, both valued by the consumer greater than their prices, the user will choose the product with the least price, and from two products, for which the user is ready to pay the same price, he will choose (if possible) the product with the greatest consumer value.

5º. The hardware and software prices are made up of fixed costs, manufacturer's profit, variable costs and technical support costs.

Fixed costs and technical support costs for the software developers are insignificant enough, and variable costs are close to zero at all (it does not cost too much to burn a CD with a copy of software product, and it costs almost nothing to release the product in the Internet).

Technical support costs for hardware and software manufacturers are approximately the same.

Fixed costs are essentially greater for hardware manufacturers than for software developers, but variable costs (as well as for software suppliers) are close to zero (because the manufacturer needs to build a hi-tech plant which costs several billion dollars in order to produce CPUs, but then the production of a CPU costs less than 1 dollar).

Products are offered in the market during quite a long time without essential changes of functionality. In other words, manufacturers incur fixed costs just once, then collect them back by manufacturing and selling products, then start to receive net profits.

Therefore it is possible to assume that *Intel*, *AMD* and *Microsoft* make their pricing decisions based on the aim of maximization of instant profits, i.e. profits calculated taking into account variable costs, but not fixed costs.

6º. Hardware and software manufacturers do not conspire and do not co-operate in other ways. Each manufacturer makes the pricing decisions based on available market information on the prices of other players' products (i. e., so-called Cournot situation is considered).

7º. When making pricing decisions each manufacturer considers that other players do not react on the change of the price by this manufacturer, i.e. cross price elasticities are equal to zero.

8º. The prices of all the products essentially exceed variable costs for these products manufacturing.

9º. PC assemblers form the market of a perfect competition, and could not affect the price of the bundled product (a PC with an operating system), unlike manufacturers of CPUs and the proprietary operating system.

It is assumed that the bundled product price is the sum of the CPU price and the operating system price.

Let's use the following designations: $q_{\max}$ — PC market capacity; C_I and C_A — the maximal possible prices for *Intel*-based PC and for *AMD*-based PC, correspondingly; C_{I+W} and C_{A+W} — maximal possible price for *Intel*-based PC running *Windows* and for *AMD*-based PC running *Windows*; c_I and c_A — prices for *Intel* and *AMD* CPUs set by manufacturers; c_W — *Windows* license price set by *Microsoft*; q_{I+W} and q_{I+L} — demand *Intel*-based PC running *Windows* and *Linux*; q_{A+W} and q_{A+L} — demand *AMD*-based PC running *Windows* and *Linux*; q_I and q_A — demand on *Intel* and *AMD* CPUs; q_W and q_L — demand on *Windows* and *Linux*; f_I, f_A and f_W — fixed costs of *Intel*, *AMD* and *Microsoft*; v_I, v_A and v_W — variable costs of *Intel*, *AMD* and *Microsoft*; π_I, π_A and π_W — profitы of *Intel*, *AMD* and *Microsoft*.

The model

In the formulated assumptions the model of hardware and software manufacturers' interaction looks as follows.

Linear demand functions are presented on fig. no. 1.

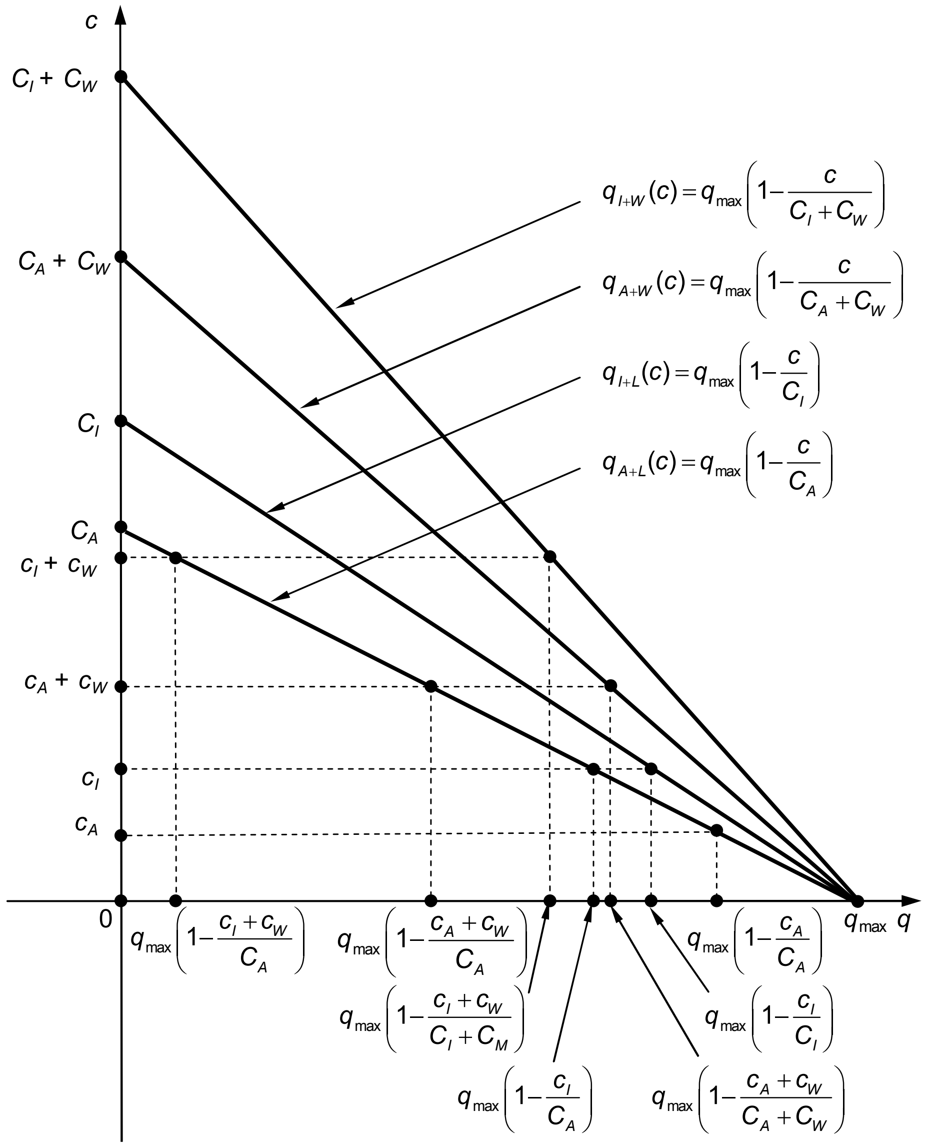

Fig. no. 1 – Linear demand functions

Thus, obviously,

$$q_I = q_{I+W} + q_{I+L}, \qquad q_A = q_{A+W} + q_{A+L};$$

$$\pi_I = q_I\left(c_I - v_I\right) - f_I, \qquad \pi_A = q_A\left(c_A - v_A\right) - f_A, \qquad \pi_W = q_W\left(c_W - v_W\right) - f_W.$$

There are two possible cases of the order of bundled products prices, not contradicting the formulated assumptions. *AMD*-based PC running *Windows* can appear more expensive, than *Intel*-based PC running *Linux*:

$$c_I + c_W > c_A + c_W > c_I > c_A, \tag{1}$$

or less expensive:

$$c_I + c_W > c_I > c_A + c_W > c_A. \quad (2)$$

Let's calculate the basic characteristics of the model for each of these cases.

In the case (1) the demand for *Intel*-based PCs running *Windows* and *Linux* will be equal to

$$q_{I+W} = q_{\max}\left(1-\left(c_I + c_W\right)/C_A\right);$$

$$q_{I+L} = q_{\max}\left(1-c_I/C_A\right) - q_{\max}\left(1-\left(c_A + c_W\right)/C_A\right) = q_{\max}\left(c_A + c_W - c_I\right)/C_A;$$

the demand for *AMD*-based PCs running *Windows* and *Linux* will be

$$q_{A+W} = q_{\max}\left(1-\left(c_A + c_W\right)/C_A\right) - q_{\max}\left(1-\left(c_I + c_W\right)/C_A\right) = q_{\max}\left(c_I - c_A\right)/C_A;$$

$$q_{A+L} = q_{\max}\left(1-c_A/C_A\right) - q_{\max}\left(1-c_I/C_A\right) = q_{\max}\left(c_I - c_A\right)/C_A.$$

respectively.

The demand for CPUs will be

$$q_I = q_{I+W} + q_{I+L} = q_{\max}\left(1-\left(c_I + c_W\right)/C_A\right) + q_{\max}\left(c_A + c_W - c_I\right)/C_A = q_{\max}\left(C_A - 2c_I + c_A\right)/C_A;$$

$$q_A = q_{A+W} + q_{A+L} = q_{\max}\left(c_I - c_A\right)/C_A + q_{\max}\left(c_I - c_A\right)/C_A = 2q_{\max}\left(c_I - c_A\right)/C_A;$$

and the demand for operating systems will be

$$q_W = q_{I+W} + q_{A+W} = q_{\max}\left(1-\left(c_I + c_W\right)/C_A\right) + q_{\max}\left(c_I - c_A\right)/C_A = q_{\max}\left(C_A - c_A - c_W\right)/C_A;$$

$$q_L = q_{I+L} + q_{A+L} = q_{\max}\left(c_A + c_W - c_I\right)/C_A + q_{\max}\left(c_I - c_A\right)/C_A = q_{\max} c_W/C_A.$$

Further calculations are given in table no. 1. The Cournot strategy for each market participant is defined as such a price of the product, which leads to the maximum profit on the assumption that the other players will not change their prices.

Table no. 1

Basic characteristics of the model

Supplier	Demand	Profit	Cournot price strategy
Case (1)			
Intel	$q_I = q_{\max}\left(C_A - 2c_I + c_A\right)/C_A$	$\pi_I = q_I\left(c_I - v_I\right) - f_I = q_{\max}\left(C_A - 2c_I + c_A\right)\left(c_I - v_I\right)/C_A - f_I$	$c_I = \left(C_A + 2v_I + c_A\right)/4$
AMD	$q_A = 2q_{\max}\left(c_I - c_A\right)/C_A$	$\pi_A = q_A\left(c_A - v_A\right) - f_A = 2q_{\max}\left(c_I - c_A\right)\left(c_A - v_A\right)/C_A - f_A$	$c_A = \left(c_I + v_A\right)/2$
Microsoft	$q_W = q_{\max}\left(C_A - c_A - c_W\right)/C_A$	$\pi_W = q_W\left(c_W - v_W\right) - f_W = q_{\max}\left(C_A - c_A - c_W\right)\left(c_W - v_W\right)/C_A - f_W$	$c_W = \left(C_A + v_W - c_A\right)/2$
Case (2)			
Intel	$q_I = q_{\max}\left(C_A - c_I - c_W\right)/C_A$	$\pi_I = q_I\left(c_I - v_I\right) - f_I = q_{\max}\left(C_A - c_I - c_W\right)\left(c_I - v_I\right)/C_A - f_I$	$c_I = \left(C_A + v_I - c_W\right)/2$
AMD	$q_A = 2q_{\max}\left(c_I + c_W - c_A\right)/C_A$	$\pi_A = q_A\left(c_A - v_A\right) - f_A = 2q_{\max}\left(c_I + c_W - c_A\right)\left(c_A - v_A\right)/C_A - f_A$	$c_A = \left(c_I + c_W + v_A\right)/2$
Microsoft	$q_W = q_{\max}\left(C_A - c_A - c_W\right)/C_A$	$\pi_W = q_W\left(c_W - v_W\right) - f_W = q_{\max}\left(C_A - c_A - c_W\right)\left(c_W - v_W\right)/C_A - f_W$	$c_W = \left(C_A + v_W - c_A\right)/2$

Obviously,

$$0 \le q_I \le q_{\max}, \qquad 0 \le q_A \le q_{\max}, \qquad 0 \le q_W \le q_{\max}.$$

For the case (1) these conditions are equivalent to the following ones:

$$2c_I - c_A \le C_A, \qquad c_A \le c_I, \qquad c_I - c_A \le C_A/2, \qquad c_A + c_W \le C_A, \quad (3)$$

and for the case (2) they are equivalent to the following conditions:

$$c_A \le c_I + c_W \le C_A, \qquad c_I + c_W - c_A \le C_A/2, \qquad c_A + c_W \le C_A. \tag{4}$$

Results

Let's assume that manufacturers change their prices in discrete time moments, and these moments coincide for all the market participants. Let $c_I^{(t)}$ and $c_A^{(t)}$ be the prices of Intel and AMD CPUs, and $c_W^{(t)}$ be Windows license prices at the time $t = 0, 1, \ldots$

Let the prices $c_I^{(0)}, c_A^{(0)}$ and $c_W^{(0)}$ at the initial moment satisfy the conditions (3) and (4) for situations (1) and (2) respectively.

It's assumed that in each moment t the manufacturer makes the pricing decision based on the known information on the prices of other products in the previous moment $t-1$.

The following assertions are fair.

Assertion 1 *on the instability of the case* (2). If at the initial time $t = 0$ the order of the prices corresponds to the case (1) then the order of the prices will change to the case (2) at the next moment $t = 1$ and then will not change any more.

Proof. If at the initial moment there was a situation (2):

$$c_I^{(0)} + c_W^{(0)} > c_I^{(0)} > c_A^{(0)} + c_W^{(0)} > c_A^{(0)},$$

then according to table no. 1

$$c_I^{(1)} = \left(C_A + v_I - c_W^{(0)}\right)/4; \qquad c_A^{(1)} = \left(c_I^{(0)} + c_W^{(0)} + v_A\right)/2; \qquad c_W^{(1)} = \left(C_A + v_W - c_A^{(0)}\right)/2.$$

Comparing $c_A^{(1)} + c_W^{(1)}$ and $c_I^{(1)}$, we obtain that

$$c_A^{(1)} + c_W^{(1)} - c_I^{(1)} = \left(2c_I^{(0)} + 3c_W^{(0)} - 2c_A^{(0)} + C_A + 2v_A + 2v_W - v_I\right)/4 > 0,$$

because $c_I^{(0)} > c_A^{(0)}$, and all the variable costs are negligible in comparison to the prices.

Thus, $c_A^{(1)} + c_W^{(1)} > c_I^{(1)}$, i. e. at the moment $t = 1$ we observe the case (1).

If at some time $t-1$ there is a situation (1):

$$c_I^{(t-1)} + c_W^{(t-1)} > c_A^{(t-1)} + c_W^{(t-1)} > c_I^{(t-1)} > c_A^{(t-1)}.$$

then according to table no. 1

$$c_I^{(t)} = \left(C_A + 2v_I + c_A^{(t-1)}\right)/4; \quad c_A^{(t)} = \left(c_I^{(t-1)} + v_A\right)/2; \quad c_W^{(t)} = \left(C_A + v_W - c_A^{(t-1)}\right)/2.$$

Comparing $c_A^{(1)} + c_W^{(1)}$ and $c_I^{(1)}$, we have the following:

$$c_A^{(1)} + c_W^{(1)} - c_I^{(1)} = \left(2\left(c_I^{(t-1)} - c_A^{(t-1)}\right) + C_A - c_A^{(t-1)} + 2\left(v_A + v_W - v_I\right)\right)/4 > 0,$$

because $c_I^{(t-1)} > c_A^{(t-1)}$, $c_A^{(t-1)} < C_A$, and all the variable costs are negligible in comparison to the prices.

The assertion is proved.

Assertion 2 *on the Cournot equilibrium prices.*

$$\lim_{t\to\infty} c_I^{(t)} = \left(2C_A + 4v_I + v_A\right)/7; \ \lim_{t\to\infty} c_A^{(t)} = \left(C_A + 2v_I + 4v_A\right)/7; \ \lim_{t\to\infty} c_W^{(t)} = \left(6C_A + 7v_W - 2v_I - 4v_A\right)/14.$$

Proof. According to table no. 1,

$$c_I^{(t)} = \left(C_A + 2v_I + c_A^{(t-1)}\right)/4 = \left(C_A + 2v_I + \left(c_I^{(t-2)} + v_A\right)/2\right)/4 = \left(C_A + 2v_I\right)/4 + \left(c_I^{(t-2)} + v_A\right)/8 =$$
$$= \left(C_A + 2v_I\right)/4 + \left(\left(C_A + 2v_I\right)/4 + \left(c_I^{(t-4)} + v_A\right)/8 + v_A\right)/8 = \left(C_A + 2v_I\right)/4 + \left(C_A + 2v_I\right)/(4\cdot 8) +$$
$$+\left(c_I^{(t-4)} + (1+8)v_A\right)/8^2 = \cdots = \left(C_A + 2v_I\right)/4 + \left(C_A + 2v_I\right)/(4\cdot 8) + \left(C_A + 2v_I\right)/\left(4\cdot 8^2\right) + \cdots +$$
$$+\left(C_A + 2v_I\right)/\left(4\cdot 8^{[t/2]-1}\right) + \left(c_I^{(t-8)} + \left(1 + 8 + 8^2 + \cdots + 8^{[t/2]-1}\right)v_A\right)/8^{[t/2]} =$$
$$= \left(C_A + 2v_I\right)\left(1 + 1/8 + 1/8^2 + \cdots + 1/8^{[t/2]-1}\right)/4 + \left(c_I^{(t-2[t/2])} + \left(1 + 8 + 8^2 + \cdots + 8^{[t/2]-1}\right)v_A\right)/8^{[t/2]} =$$
$$= \left(2C_A + 4v_I + v_A\right)\left(1 - 1/8^{[t/2]}\right)/7 + c_I^{(t-2[t/2])}/8^{[t/2]},$$

whence

$$\lim_{t\to\infty} c_I^{(t)} = \lim_{t\to\infty}\left(\left(2C_A + 4v_I + v_A\right)\left(1 - 1/8^{[t/2]}\right)/7 + c_I^{(t-2[t/2])}/8^{[t/2]}\right) = \left(2C_A + 4v_I + v_A\right)/7.$$

Similarly,

$$c_A^{(t)} = \left(c_I^{(t-1)} + v_A\right)/2 = \left(\left(C_A + 2v_I + c_A^{(t-2)}\right)/4 + v_A\right)/2 = \left(C_A + 2v_I + 4v_A + c_A^{(t-2)}\right)/8 =$$
$$= \left(C_A + 2v_I + 4v_A\right)/8 + c_A^{(t-2)}/8 = \left(C_A + 2v_I + 4v_A\right)/8 + \left(C_A + 2v_I + 4v_A\right)/8^2 + c_A^{(t-2)}/8^2 = \cdots =$$
$$= \left(C_A + 2v_I + 4v_A\right)/8 + \left(C_A + 2v_I + 4v_A\right)/8^2 + \cdots + \left(C_A + 2v_I + 4v_A\right)/8^{2[t/2]} + c_A^{(t-2[t/2])}/8^{2[t/2]} =$$
$$= \left(C_A + 2v_I + 4v_A\right)\left(1 - 1/8^{2[t/2]}\right)/7 + c_A^{(t-2[t/2])}/8^{2[t/2]};$$
$$c_W^{(t)} = \left(C_A + v_W - c_A^{(t)}\right)/2 = \left(C_A + v_W - \left(C_A + 2v_I + 4v_A\right)\left(1 - 1/8^{2[t/2]}\right)/7 - c_A^{(t-2[t/2])}/8^{2[t/2]}\right)/2 =$$
$$= \left(6C_A + 7v_W - 2v_I - 4v_A\right)/14 + \left(C_A + 2v_I + 4v_A\right)/\left(14\cdot 8^{2[t/2]}\right) - c_A^{(t-2[t/2])}/\left(2\cdot 8^{2[t/2]}\right).$$

whence

$$\lim_{t\to\infty} c_A^{(t)} = \lim_{t\to\infty}\left(\left(C_A + 2v_I + 4v_A\right)\left(1 - 1/8^{2[t/2]}\right)/7 + c_A^{(t-2[t/2])}/8^{2[t/2]}\right) = \left(C_A + 2v_I + 4v_A\right)/7;$$
$$\lim_{t\to\infty} c_W^{(t)} = \lim_{t\to\infty}\left(\left(6C_A + 7v_W - 2v_I - 4v_A\right)/14 + \left(C_A + 2v_I + 4v_A\right)/\left(14\cdot 8^{2[t/2]}\right) - c_A^{(t-2[t/2])}/\left(2\cdot 8^{2[t/2]}\right)\right) =$$
$$= \left(6C_A + 7v_W - 2v_I - 4v_A\right)/14,$$

QED.

If we assume that instant profit is the difference between gain and variable costs, then the following assertion is fair.

Assertion 3 *on equilibrium instant profit of market players.*

$$\lim_{t\to\infty} \tilde{\pi}_I^{(t)} = \lim_{t\to\infty}\left(q_{\max}\left(C_A - 2c_I^{(t)} + c_A^{(t)}\right)\left(c_I^{(t)} - v_I\right)/C_A\right) = 2q_{\max}\left(\left(2C_A - 3v_I + v_A\right)/7\right)^2/C_A;$$
$$\lim_{t\to\infty} \tilde{\pi}_A^{(t)} = \lim_{t\to\infty}\left(2\,q_{\max}\left(c_I^{(t)} - c_A^{(t)}\right)\left(c_A^{(t)} - v_A\right)/C_A\right) = 2q_{\max}\left(\left(C_A + 2v_I - 3v_A\right)/7\right)^2/C_A;$$
$$\lim_{t\to\infty} \tilde{\pi}_W^{(t)} = \lim_{t\to\infty}\left(q_{\max}\left(C_A - c_A^{(t)} - c_W^{(t)}\right)\left(c_W^{(t)} - v_W\right)/C_A\right) = q_{\max}\left(\left(6C_A - 2v_I - 4v_A - 7v_W\right)/14\right)^2/C_A.$$

Proof is conducted by substitution of limit values and into the expressions for the instant profits.

Conclusions

If we ignore variable costs, then

$$\lim_{t\to\infty} c_I^{(t)} \approx 2C_A/7;\ \lim_{t\to\infty} c_A^{(t)} \approx C_A/7;\ \lim_{t\to\infty} c_W^{(t)} \approx 3C_A/7;$$
$$\lim_{t\to\infty}\left(c_I^{(t)} + c_W^{(t)}\right) \approx 5C_A/7;\quad \lim_{t\to\infty}\left(c_A^{(t)} + c_W^{(t)}\right) \approx 4C_A/7;$$
$$\lim_{t\to\infty} \tilde{\pi}_I^{(t)} \approx 8q_{\max}C_A/49;\qquad \lim_{t\to\infty} \tilde{\pi}_A^{(t)} \approx 2q_{\max}C_A/49;\qquad \lim_{t\to\infty} \tilde{\pi}_W^{(t)} \approx 9q_{\max}C_A/49.$$

We can see that the most expensive bundle product (an *Intel*-based PC running *Windows*) in the Cournot equilibrium is approximately 5 times more expensive than the cheapest product (*AMD*-based PC running *Linux*); an *Intel* CPU costs approximately 2 times more than *AMD* processor; *Windows* license is approximately 1,5 times more expensive than Intel processor, and approximately 3 times more expensive than *AMD* processor.

The instant profit of *Intel* is 4 times greater than the instant profit of *AMD*, while the instant profit of *Microsoft* is just 12,5% greater than the instant profit of *Intel*.

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